The Banach Center Conference

30 YEARS OF BI-HAMILTONIAN SYSTEMS

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"In the late 1970's, two fundamental concepts appeared in the theory of integrable systems, recursion operators (sometimes called strong symmetries or Λ-operators) that map symmetries to symmetries or, dually, conservation laws to conservation laws, and bi-Hamiltonian structures. The latter notion is due to Magri (1978) though Gelfand and Dorfman introduced it independently a year later under the name of 'Hamiltonian pairs'. Examples of bi-Hamiltonian structures had been discovered by Adler, and by Gelfand and Dikii in a 1978 paper which was never published in Russian but was finally published, in English translation, in 1987 in the Collected Papers of Gelfand (volume 1) though it is still often cited as a preprint, as is the case in the book under review. The operator, obtained by composing one Hamiltonian mapping with the inverse of another (assumed to be invertible) is a recursion operator for any bi-Hamiltonian system, i.e. vector field which is Hamiltonian with respect to both structures. The Hamiltonian or Poisson structures of a Hamiltonian pair satisfy a compatibility condition which implies that the Nijenhuis torsion of this operator vanishes. This important fact and its applications are to be found in the first (1979) of the series of papers published by Gelfand and Dorfman in Funct. Anal. Appl. while it was discovered independently and at the same time by Magri and emphasized by Fokas and Fuchssteiner. From then on the theory of bi-Hamiltonian systems and its applications to integrable systems developed rapidly."

Yvette Kosmann-Schwarzbach

(From the review of the book "Dirac Structures and Integrability of Nonlinear Evolution Equations" by Irene Dorfman.)

Basak Gancheva

Explicit solution for the Zhukovski-Volterra gyrostat

Inna Basak Gancheva, Universidad Politécnica de Cataluña

Abstract

This is an analytical study of the simple classical generalization of the Euler top problem: the Zhukovski–Volterra (ZV) system describing the free motion of a gyrostat (a rigid body carrying arotator inside), which was first investigated by N. Zhukovski and, independently, by V. Volterra (1899). In contrast to the Euler top, the equations of motion of ZV are not homogeneous, which makes their integration technically more complicated. We revise the solutions for the angular momentum first obtained by Volterra and present alternative solutions based on an algebraic parametrization of the invariant curves . This also enables us to derive an effective description of the motion of the body in space. The proposed construction is completely explicit and requires resolving three quartic algebraic equations.

Błaszak

Bi-Hamiltonian Property and related class of separation curves

Maciej Błaszak, Adam Mickiewicz University, Poznań

Abstract

It is shown that separation conditions are fundamental objects of separability theory. They can be used for the classification of separable systems. Next, it is shown that for certain classes of separable systems the bi-Hamiltonian extension is a natural property. Finally it is shown that considered classes of separable systems are related to each other by generalized Stackel transforms on the level of constants of motion and by multi-times reciprocal transformations on the level of equations of motion.

Bolsinov

Singular Lagrangian Fibrations and bi-Hamiltonian systems

Alexey Bolsinov, Loughborough University

Abstract

A singular Lagrangian fibration associated with an integrable Hamiltonian system on a Poisson manifold M is the decomposition of M into common levels

$$X_a = \{f_1 = a_1, f_2 = a_2, \ldots, f_l = a_l\}$$

of commuting first integrals $f_1, f_2, \ldots, f_l : M \to \mathbb{R}^l$. The topology of such a fibration is closely related to qualitative properties of the system and contains the information about bifurcations of Liouville tori, types of equilibrium points, stable and unstable closed trajectories, etc. The purpose of the talk is to discuss how bi-Hamiltonian structure of a given system may help us in topological analysis of the corresponding Lagrangian fibration. As an example we consider integrable tops on semisimple Lie algebras to show how natural the reformulation of some topological phenomena in terms of a pencil of compatible Poisson bracket is. It seems that this relationship is not quite understood yet and we would like to formulate some open questions.

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Casati

The Bi-Hamiltonian structures of multi-components KdV-type hierarchies

Paolo Casati, University of Milano-Bicocca

Abstract

In this talk we shall consider hierarchies of multi–components KdV–type equations. We shall present their bi– and multi–Hamiltonian structures and their lax formulations. Further, in the first non–trivial case it will be explicitly shown how the entire hierarchy may be recovered by its bi–Hamiltonian formulation. Finally some properties of their stationary reductions are also deduced.

(Joint work with Giacomo Galetti and Giovanni Ortenzi)

Falqui

Gaudin systems and their limits: classical and quantum cases

Gregorio Falqui, University of Milano-Bicocca

Abstract

We will (re)consider families of bi-Hamitonian structures on direct sums of Lie algebras, and in particular, the (bi)-Poisson and quantum algebras of the mutually commuting constants of the motion associated with Gaudin systems. We will discuss the limit when (some of) the poles of the Lax matrix coincide. We shall relate these algebras to the Hamiltonians of the Kapovich-Millson Bending Flows. We shall show how, using a suitable version of Talalaev's determinantal formula, as well as the notion of Manin matrix, one can explicitly define such quantum integrals in the limiting case.

Federov

Continuous and discrete Neumann systems on Stiefel varieties

Yuri Federov, Universidad Politécnica de Cataluña

Abstract

We consider natural generalizations of the Neumann systems on Stiefel varieties $V(n,r)$ with the Eulidean and normal standard metric and prove their integrability in the non-commutative sense by presenting compatible Poisson brackets on $T^*V(n,r)$. Apart from known Lax pair for the generalized Neumann systems given by Reyman and Semenov, an alternative Lax pair is presented, which enables us go give a complete algebraic geometric description of the systems, as well as to construct their integrable discretizations. This talk is the result of a joint work with Bozidar Jovanovic.

Hentosh

On a Hamiltonian nonlocal finite-dimensional reduction of the Lax-integrable supersymmetric Davey-Stewartson system

Oksana Hentosh, Institute of Mechanics and Mathematics, NAS, Ukraine

Abstract

Jurdjevic

Integrable Hamiltonian systems on symmetric spaces

Velimir Jurdjevic, University of Toronto

Abstract

In this lecture I will discuss certain variational problems on a Lie group G whose Lie algebra $\mathfrak{g}$ admits a Cartan decomposition

$$\mathfrak{g} = \mathfrak{p} + \mathfrak{k} \qquad , with$$
$$[\mathfrak{p}, \mathfrak{p}] \subseteq \mathfrak{k} \quad , \quad [\mathfrak{p}, \mathfrak{k}] \subseteq \mathfrak{p}, \quad [\mathfrak{k}, \mathfrak{k}] \subseteq \mathfrak{k}.$$

Typically such a decomposition is induced by an involutive automorphism σ on a Lie group G with $\mathfrak{k}$ equal to the sub-algebra of fixed points of the tangent map σ_* of σ and $\mathfrak{p} = \{A \in \mathfrak{g} : \sigma_*(A) = -A\}$.

I will show the effectiveness and relevance of optimal control theory for the geometry of the associated symmetric space $M = G/K$ by considering the problem of minimizing the functional $\frac{1}{2}\int_0^T ||U(t)||^2 dt$ over the trajectories $g(t)$ in G of a left invariant differential system

$$\frac{dg}{dt} = g(A + U(t))$$

where the controls $U(t)$ are absolutely continuous curves that take values in $\mathfrak{k}$. The norm on $U(t)$ is the norm induced by the Cartan-Killing form and A is a fixed matrix in $\mathfrak{p}$ such that for any g_0 and g_1 in G there is a control $U(t)$ that generates the trajectory $g(t)$ with $g(0) = g_0$ and $g(T) = g_1$. Under these assumptions the above problem is well defined for each pair of boundary values g_0 and g_1.

The Maximum principle shows that the optimal curves are the projections of the integral curves of a Hamiltonian H on the dual $\mathfrak{g}^*$ of $\mathfrak{g}$. We will show that this Hamiltonian is always integrable and we will also discuss various aspects of its solutions in relation to the Euler-Griffiths elastic problem on $M = G/K$, Moser's isospectral problems inspired by Jacobi's geodesic problem on an ellipsoid and optimal transfer problems in quantum control.

Lenells

The Hunter-Saxton equation describes the geodesic flow on a sphere

Jonatan Lenells, Cambridge University

Abstract

The Hunter-Saxton equation is an integrable bi-Hamiltonian equation modeling the propagation of orientation waves in a liquid crystal director field. It describes the geodesic flow on the infinite-dimensional homogeneous space $\mathrm{Diff}(S^1)/S^1$ with respect to a certain right-invariant metric. We show that this space has constant positive curvature and is in fact isometric to a subset of the L^2 unit sphere. We explain how this geometric picture naturally leads to a construction of global weak solutions of the equation.

Maciejewski

Necessary conditions for super and partial integrability of Hamiltonian systems

Andrzej Maciejewski, University of Zielona Góra

Abstract

We formulate a general theorem which gives a necessary condition for the maximal super-integrability of a Hamiltonian system. This condition is expressed in terms of properties of the differential Galois group of the variational equations along a particular solution of the considered system. An application of this general theorem to natural Hamiltonian systems of n degrees of freedom with a homogeneous potential gives easily computable and effective necessary conditions for the super-integrability. To illustrate an application of the formulated theorems, we investigate: three known families of integrable potentials, and the three body problem on a line.

We found also necessary conditions for partial integrability of Hamiltonian systems and we illustrate their applications investigating several systems.

(joint work with Maria Przybylska and Haruo Yoshida)

Magri

Fragments of Bihamiltonian Geometry

Franco Magri, University of Milano-Bicocca

Abstract

I discuss two variations on the theme of recursion operator, one old and one modern. The first, related to Lagrange, signs the birth of bihamiltonian geometry. The second, related to the theory of Frobenius manifolds, shows the great versatility of this concept.

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Marle

Dirac brackets and bi-Hamiltonian structures

Charles-Michel Marle, Unversité Pierre et Marie Curie, Paris

Abstract

Around 1950, P. Dirac developed a *Generalized Hamiltonian dynamics* for Lagrangian systems with a degenerate Lagrangian. In this theory, the phase space of the system (i.e. the cotangent bundle to the configuration manifold) is endowed with two Poisson brackets:

- the usual Poisson bracket associated to its symplectic structure,
- a modified Poisson bracket (today known as the *Dirac bracket*), used by Dirac for the canonical quantization of the system.

I will describe Dirac's theory of generalized Hamiltonian systems, and further developments due to Bergmann, Gotay and Nester, Tulczyjew, in the language of modern symplectic geometry. I will present several examples, and I will discuss its links with the theory of bihamiltonian systems.

Marmo

Quantum and classical bi-Hamiltonian Systems

Giuseppe Marmo, Universita Federico II di Napoli

Abstract

The report is divided in four main parts.

1)From the equations of motion to the canonical commutation relations:A brief historical excursus.

2)Quantum mechanics on phase space :the Wigner-Weyl picture

3)The quantum-classical transition.

4)The classical limit of quantum biHamiltonian systems:problems and perspectives.

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Misiolek

Bihamiltonian equations on the Virasoro group

Gerard Misiolek, University of Notre Dame

Abstract

A number of interesting nonlinear PDE arise as geodesic equations on the group of diffeomorphisms of the circle. I will describe one such equation and discuss its bihamiltonian structure and wellposedness.

Mykytyuk

The "Stages-Hypothesis", reduction by stages an structure of coadjoint orbits of Lie groups

Ihor V. Mykytyuk, Rzeszów University, Poland; IAPMM NAS, Ukraine

Abstract

A symplectic reduction may be described as follows: given the symplectic action of a Lie group on a symplectic manifold having a momentum map, one divides a level set of the momentum map by the action of a suitable subgroup to form a new symplectic manifold. Before the division step, one has a manifold carrying a degenerate closed 2-form. Removing such a degeneracy by passing to a quotient space one obtains the reduced symplectic manifold [1, 2].

The issue of performing reduction by stages arises already in the paper of Marsden and Weinstein [1] and may be formulated as follows: one wants a framework in which repeated reduction by two successive symmetry groups can be performed and the result is the same as that of a single larger group ([2]). One of the nicest examples of reduction by stages is the theory of semidirect product reduction that is due to Guillemin and Sternberg [3] and Marsden, Ratiu and Weinstein [4, 5].

The issue of performing reduction by stages in its final and most general form was formulated by Marsden and Ratiu (see [2]). They decided that the framework of starting with a big group M with a normal subgroup N and trying to reduce first by N and then by some kind of a quotient group M_ν/N_ν (not exactly by M/N, $M_\nu \subset M$, $N_\nu \subset N$) was the right framework for reduction by stages theory.

In this talk we represent a short proof [7] of the Stages Hypothesis of Marsden-Misiołek-Ortega-Perlmutter-Ratiu (MMOPR), which is a sufficient condition for a general reduction by stages theorem [6],[2]. In the book [2] this hypothesis was verified for all split group extensions M of a Lie group N. In particular, both central extensions and semidirect products with a vector space fit into this class. We give the short Lie-algebraic proof of this hypothesis in a general case, for arbitrary pair (M, N) of a Lie group M and its normal not-necessary closed subgroup N.

Our proof of the hypothesis is based on changing of the approach and the point of view: reformulating the Stages Hypothesis we obtain this hypothesis as a general fact in the structure theory of co-adjoint orbits of Lie groups: each M-coadjoint orbit contains some affine subspace determined by the normal subgroup N and this subspace is a N_ν-orbit.

Moreover, we solve the non-equivariance problem arising in [2]: starting with an equivariant moment map J for the Lie group M, trying to first reduce by N (to obtain the first reduced space) and then by the quotient group M_ν/N_ν (to obtain the second reduced space), in general the action of this quotient group on the first reduced space is non-Hamiltonian, i.e. the corresponding moment map for the group M_ν/N_ν induced by J is non-equivariant. We solve this problem replacing the quotient group M_ν/N_ν by the group M_ν which acts equivariantly and the corresponding quotient space (second reduced space) is the same since the subgroup N_ν acts trivially. Remark that in this case we use weaker conditions to carry out the reduction by stages procedure. Moreover, as a corollary in the coadjoint orbits theory we obtain one-to-one correspondence between integral coadjoint orbits of the Lie group M and integral coadjoint orbits of the Lie groups, which are one-dimensional central extensions of M_ν/N_ν.

References

[1] Marsden J., Weinstein A., Reduction of symplectic manifolds with symmetry, *Rep. Math. Phys.*, **5** (1974), 121–130.

[2] Marsden J., Misiołek G., Ortega J., Perlmutter M., Ratiu T., *Hamiltonian Reduction by Stages*, Lecture Notes in Mathematics, **1913**, 2007, XV, 519 p.

[3] Guillemin V., Sternberg S., The moment map and collective motion, *Ann. of Phys.*, **1278** (1980), 220–253.

[4] Marsden J., Ratiu T., Weinstein A., Semidirect products and reduction in mechanics, *Trans. Amer. Math. Soc.*, **281** (1984), 147–177.

[5] Marsden J., Ratiu T., Weinstein A., Reduction and Hamiltonian structures on duals of semidirect product Lie Algebras, *Contemp. Math.*, Am. Math. Soc., **28** (1984), 55–100.

[6] Marsden J., Misiołek G., Perlmutter M., Ratiu T., Symplectic reduction for semidirect products and central extensions, *Diff. Geom. and its Appl.*, **9** (1998), 173–212.

[7] Mykytyuk I., Stepin A., Sufficient condition of reduction by stages: the proof and applications, *Mat. Sb*, **199**:5 (2008), 35–44; Engl. transl. in Sb. Math. **199**:5 (2008).

Ovsienko

Non-linear bi-Hamiltonian systems in dimension 2+1 and looped cotangent Virasoro algebra

Valentin Ovsienko, University of Lyon 1

Abstract

We study non-linear integrable partial differential equations naturally arising as bi-Hamiltonian Euler equations related to the looped cotangent Virasoro algebra. This infinite-dimensional Lie algebra (recently constructed by Claude Roger and the author) is a generalization of the classical Virasoro algebra to the case of two space variables. Among the examples of bi-Hamiltonian Euler equations, there is so-called "universal equation" which is quite well known in the physical and mathematical literature.

Panasyuk

Bi-Hamiltonian structures from the point of view of symmetries

Andriy Panasyuk, University of Warsaw

Abstract

There are two classical ways of constructing integrable systems by means of bihamiltonian structures. The first one supposes nondegeneracy of one of the Poisson structures generating the pencil and uses the so-called recursion operator. This situation corresponds to the absence of Kronecker blocks in the Gelfand-Zakharevich decomposition. The second one, which corresponds to the absence of Jordan blocks in this decomposition, uses the Casimir functions of different members of the pencil.

Alexei Bolsinov studied examples of bihamiltonian structures coming from the so-called Lie pencils in which both the Kronecker and Jordan blocks are present in the Gelfand-Zakharevich decomposition. He gave some sufficient conditions for the completeness of the corresponding family of functions in involution.

In this talk we shall discuss the general case of a bihamiltonian structure with Kronecker and Jordan blocks and give a criterion of the completeness of the corresponding family of functions both on the linear-algebraic and geometric levels. This criterion is related to a natural action of some Lie algebra which gives a symmetry of the whole pencil.

The criterion is applied to Lie pencils, producing more general sufficient conditions of completeness. We shall also discuss other possible applications.

Pavlov

New Hamiltonian formalism and Lagrangian representations for integrable hydrodynamic type systems

Maxim Pavlov, P.N. Lebedev Physical Institute RAS, Moscow

Abstract

New Hamiltonian formalism based on the theory of conjugate curvilinear coordinate nets is established. All formulas are mirrored to corresponding formulas in the Hamiltonian formalism constructed by B.A. Dubrovin and S.P. Novikov (in a flat case) and E.V. Ferapontov (in a non-flat case). In the mirrored-flat case Lagrangian formulation is found. Multi-Hamiltonian examples are presented. In particular Egorovs case, generalizations of local NutkuOlvers Hamiltonian structure and corresponding Sheftel-Teshukovs recursion operator are presented. An infinite number of local Hamiltonian structures of all odd orders is found.

Przybylska

Progress in the integrability studies of Hamiltonian systems with three degrees of freedom

Maria Przybylska, Nicolaus Copernicus University, Toruń

Abstract

We consider natural complex Hamiltonian system with n degrees of freedom given by a Hamiltonian function which is a sum of the standard kinetic energy and a homogeneous polynomial potential V of degree $k > 2$. The well known Morales-Ramis theorem gives the strongest known necessary conditions for the Liouville integrability of such systems. It states that for each k there exists an explicitly known infinite set $\mathcal{M}_k \subset \mathbb{Q}$ such that if the system is integrable, then all eigenvalues of the Hessian matrix $V''(\boldsymbol{d})$ calculated at a non-zero $\boldsymbol{d} \in \mathbb{C}^n$ satisfying $V'(\boldsymbol{d}) = \boldsymbol{d}$, belong to $\mathcal{M}_k$.

The aim of this paper is, among others, to sharpen this result. Under certain genericity assumption concerning V we prove the following fact. For each k and n there exists a finite set $\mathcal{I}_{n,k} \subset \mathcal{M}_k$ such that if the system is integrable, then all eigenvalues of the Hessian matrix $V''(\boldsymbol{d})$ belong to $\mathcal{I}_{n,k}$. We give an algorithm which allows to find sets $\mathcal{I}_{n,k}$.

We applied this results for the case $n = k = 3$ and we found all integrable potentials satisfying the genericity assumption. Among them several are new and they are integrable in a highly non-trivial way. We found three potentials for which the additional first integrals are of degree 4 and 6 with respect to the momenta.

Also nongeneric cases are analysed and their integrability analysis is shown. Forms of potentials in these cases are related to normal forms of the Hessian $V''(\boldsymbol{d})$ calculated at a $\boldsymbol{d} \in \mathbb{C}^n$ satisfying $V'(\boldsymbol{d}) = 0$.

Rauch-Wojciechowski

Transport of bi-Hamiltonian structures from soliton equations to separable systems of potential and quasi-potential Newton equations

Stefan Rauch-Wojciechowski, Linköping University, Sweden

Abstract

The KdV and Harry Dym hierarchies of soliton equations, which are related to the Schrdinger spectral problem, admit a bihamiltonian structure rendering all these equations to be integrable. The stationary flows and restricted flows (describing the KdV fields coupled to square eigenfunctions) are finite dimensional Hamiltonian systems invariant w.r.t. all flows of the hierarchy and are, therefore, also completely integrable. In 1990 we have found (together with M.Antonowicz) a way of transporting the bihamiltonian structure of the KdV hierarchy into restricted flows. This led us to discovery of bihamiltonian structures for the Garnier system, separable potentials and for the quasipotential Newton equations.

In 1992 I have found that all stationary flows and restricted flows of the KdV and of the Harry Dym hierarchies admit a new type of parametrisation as a set of Newton equations of potential type with definite or indefinite kinetic energy term. This discovery became an origin of theory of quasipotential Newton equations that has been developed in Linkping. This theory generalizes the classical Hamilton-Jacobi separability of separability natural Hamiltonian systems and leads to many new types of separation variables defined by sets of (non-confocal quadrics).

Respondek

Integrability properties of the geodesic equation in sub-Riemannian spaces

Witold Respondek, INSA de Rouen, France

Abstract

In our talk we will study integrability properties of the geodesic equation for sub-Riemannian spaces $(M, \mathcal{D}, B)$ where M is a smooth manifold, $\mathcal{D}$ is a smooth distribution on M and B is a smoothly varying positive definite quadratic form on $\mathcal{D}$.

In the first part we will deal with 3-dimensional homogeneous sub-Riemannian spaces such that the distribution $\mathcal{D}$ is a contact structure on M. Our main result shows deep relations between the geometry of the sub-Riemannian space and integrability properties of its geodesic equation:

Theorem 1 *For a given sub-Riemannian homogeneous space, the following conditions are equivalent:*

(i) *The sub-Riemannian space is symmetric.*
(ii) *The adjoint geodesic equation has two functionally independent quadratic first integrals;*
(iii) *The optimal controls, of the related optimal control problem, are elliptic functions;*
(iv) *All solutions of the complexified adjoint geodesic equation are single-valued functions of the complex time.*

For homogeneous, but non symmetric, sub-Riemannian spaces, the adjoint geodesic equation admits a rational (in particular, a polynomial but never of degree two) fist integral independent with the hamiltonian only for a countable set of the classification parameter. For all other values, the adjoint geodesic equation does not admit any real-meromorphic first integral independent with the hamiltonian.

Then we consider still 3-dimesional but non homogenous sub-Riemannian spaces and their nilpotent approximations. We prove that the geodesic equation in the nilpotent tangent case of the 3-dimensional sub-Riemannian problem is never integrable, except for two particular values of the classification parameter.

Finally, we quit the 3-dimensional case and consider sub-Riemannian problems on $SO(n)$ defined by a distribution of rank $n-1$ (actually those sub-Riemannian problems describe the optimal problem of minimizing the energy of transfer pulses in n-level quantum systems). For $n = 3$, the problem is integrable in terms of trigonometric functions (as one of the cases appearing in Theorem 1 above). This changes completely for $n \geq 4$. Namely, using the Morales-Ramis theory we prove:

Theorem 2 *For the n-level system on $SO(n)$, $n \geq 4$, the complexification of the adjoint geodesic equation is not integrable in the meromorphic category. More precisely, restricted to the leaves of the symplectic foliation, does not possess any meromorphic first integral independent of the hamiltonian, i.e., is not Liouville integrable on the leaves.*

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Sarlet

Decoupling of second-order equations and quasi-bi-Hamiltonian systems

Willy Sarlet, Ghent University, Belgium

Abstract

We first discuss the geometry of partially decoupling (submersive) second-order equations in general. In the case of Lagrangian systems of mechanical type, it is shown that submersiveness implies separation into subsystems. A more interesting case is that of non-conservative systems of cofactor type, as introduced by Lundmark and Wojciechowski: we explain how their work on so-called 'driven cofactor systems' generalizes from a Euclidean space to an arbitrary Riemannian one. The reduced or 'driven' part of such a system is a cofactor pair system in its own right and presents itself, after a time-rescaling, as a quasi-bi-Hamiltonian system which is Hamilton-Jacobi separable in adapted coordinates. To finish, we initiate a kind of generalized form of submersiveness of second-order systems.

Sergyeyev

Generalized Stäckel Transform: Integrability, Reciprocal Transformations and More

Artur Sergyeyev, Silesian University, Czech Republic

Abstract

We present a multiparameter generalization of the Stäckel transform, also known as the coupling-constant metamorphosis, and show that under certain conditions this generalized Stäckel transform preserves Liouville integrability, noncommutative integrability and superintegrability. The corresponding transformation for the equations of motion proves to be nothing but a reciprocal transformation of a special form, and we investigate the properties of this class of reciprocal transformations, including applications to hydrodynamic-type systems.

This is joint work with Maciej Błaszak.

References

[1] A. Sergyeyev, M. Błaszak, Generalized Stäckel Transform and Reciprocal Transformations for Finite-Dimensional Integrable Systems, J. Phys. A: Math. Theor., **41** (2008), paper 105205; preprint arXiv:0706.1473 [nlin.SI] (arXiv.org)

[2] M. Błaszak, A. Sergyeyev, A Coordinate-Free Construction for a Class of Integrable Hydrodynamic-Type Systems, preprint arXiv:0803.0308 [nlin.SI] (arXiv.org)

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Silindir Yantir

Bi-Hamiltonian structures for integrable systems on time scales

Burcu Silindir Yantir, Adam Mickiewicz University, Poznań

Abstract

In order to embed the study of integrable systems into a more general unifying framework, one of the possible approaches is to construct the integrable systems on time scales. We apply the R-matrix formalism to the algebra of δ-differential operators in terms of which one can construct infinite hierarchy of commuting vector fields. We introduce a trace formula on time scales, construct the form of linear Poisson tensors and related Hamiltonians. The construction of the quadratic Poisson tensor is given by the use of recursion operators of the Lax hierarchies. The theory is illustrated by finite-field integrable hierarchy on time scales which is Δ-differential counterpart of AKNS.

(This is a joint work Błażej SzablikowskiMaciej Błaszak)

Skrypnyk

Special quasigraded Lie algebras, compatible Poisson brackets and integrable Hamiltonian systems

Taras Skrypnyk, BITP, Ukraine

Abstract

We review a series of recent results on the special quasigraded (almost-graded) infinite-dimensional Lie algebras that admit Kostant-Adler-Symes scheme. We outline their connections with the theory of compatible Poisson brackets and classical r-matrices. We show how to construct, using them a wide class of finite-dimensional Hamiltonian systems admitting Lax representations. Among the constructed systems are different generalizations of the Steklov-Liapunov, Steklov-Veselov, Clebsch and other "anisotropic" systems.

Strachan

From flat pencils to Frobenius manifolds and hyperplane arrangements

Ian Strachan, University of Glasgow

Abstract

From the geometry of flat pencils of metrics one obtains, via the Dubrovin-Novikov theorem on 1st-order Hamiltonian structures, bi-Hamiltonian structures and geometric structures on certain cotangent bundles. With some natural extra structures (e.g. homogeneity) one obtains a Frobenius manifold.

A curved version of this construction is presented which encodes geometrically the properties of certain non-local bi-Hamiltonian structures. Examples of this type are given, including families coming from hyperplane arrangements associated to Coxeter groups and their discriminants.

(This is joint work with L. David)

Szablikowski

Central extensions of universal hierarchy: (2+1)-dimensional bi-Hamiltonian systems

Błażej Szablikowski, Adam Mickiewicz University, Poznań

Abstract

We consider the R-matrix formalism on the semi-direct product of the loop algebra of vector fields on circle and its dual. On this "double" algebra one can introduce natural ad-invariant symmetric bilinear product. Through central extensions, given by appropriate two-cocycles, additional independent variable as well as dispersion are introduced. As a result (2+1)-dimensional integrable hierarchies together with their (explicit) bi-Hamitonian structures (without operand formalism) are constructed. Examples will be provided.

(It is a joint work with Artur Sergyeyev)

Tiglay

Degasperis-Procesi Family and Density Modules

Feride Tiglay, University of New Orleans

Abstract

We will show the derivation of the Degasperis-Procesi family of equations as a Arnold-Euler equation on an extension of Diff($\mathbb{T}$) by density modules. Both Camassa-Holm and Degasperis-Procesi equations are in this family and the derived equations are two component generalizations that reduce to the equations in the Degasperis-Procesi family under certain identifications.

Turiel

About the product theorem in bi-Hamiltonian structures

Francisco-Javier Turiel, University of Málaga

Abstract

Roughly speaking a couple of bivector, on a finite dimensional real or complex space, is always the product of Kronecker factor and a symplectic one; to the last factor one associates a morphism and its characteristic polynomial.

Let $\{\Lambda, \Lambda_1\}$ be a bi-hamiltonian structure on a real analytic or complex (holomorphic) manifold M. In this talk we show that, around each point of a dense open set of M, $\{\Lambda, \Lambda_1\}$ splits into a product of a Kronecker bi-hamiltonian structure and a symplectic one provided that a well necessary condition, on the characteristic polynomial of the algebraic symplectic factor, holds.

Zakharevich

On spectral theory of operator pencils $A + tB : V \mapsto W$

Ilya Zakharevich, University of California Berkeley

Abstract

At a point of a bihamiltonian integrable system, two Hamiltonian operators A,B are, typically, two (pseudo-)differential operators. A spectral theory of an operator in a finite-dimensional vector space is completely determined by its Jordan decomposition. A pencil A + tB is a generalization of an operator; it may encode more complicated data of linear algebra, such as partially defined operators, a 1-to-many operators, etc.

For finite-dimensional pencils one must, in addition to Jordan blocks, consider Kronecker blocks. A similar thing happens in the case of (pseudo-)differential pencils. However, while finite-dimensional Kronecker blocks have only discrete parameters, in infinite-dimensional case, such blocks acquire parameters, which carry a semantic of "fuzzy eigenvalues".

Zhang

Central invariants for the Drinfeld-Sokolov bi-Hamiltonian structures

Youjin Zhang, Tsinghua University

Abstract

Central invariants of semisimple bihamiltonian structures appear in the study of the problem of classification of bihamiltonian integrable PDEs, they provide a set of invariants of semisimple bihamiltonian structures under Miura-type transformations. We review in this talk some results on properties of such invariants and consider in particular the central invariants of the Drinfeld-Sokolov bihamiltonian structures.

Notes

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